[8]Bamieh, B. A., and Pearson, J. B., Jr., "A General Framework for Linear Periodic Systems with Applications to $H_\infty$ Sampled-Data Control," *IEEE Transactions on Automatic Control*, Vol. 37, No. 4, 1992, pp. 418–435.

[9]Balas, G. J., Doyle, J. C., Glover, K., Packard, A., and Smith, R., *μ-Analysis and Synthesis Toolbox*, MUSYN, Inc., and Math Works, Inc., Natick, MA, 1993.

[10]Zhou, K., Doyle, J. C., and Glover, K., *Robust and Optimal Control*, Prentice–Hall, Upper Saddle River, NJ, 1996, Chaps. 17–19.

[11]Lorenzo, C. F., "Continuum Fatigue Damage Modeling for Use in Life Extending Control," NASA TM 106691, Aug. 1994.

[12]Gill, P. E., Murray, W., and Wright, M. H., *Practical Optimization*, Academic Press, London, 1981, Chap. 6.

[13]Lorenzo, C. F., Holmes, M. S., and Ray, A., "Design of Life Extending Controls Using Nonlinear Parameter Optimization," NASA TP 3700, April 1998.

[14]Khalil, H. K., *Nonlinear Systems*, 2nd ed., Prentice–Hall, Upper Saddle River, NJ, 1996, Chaps. 11–13.

# Output Feedback Variable Structure Control for Uncertain Systems with Input Nonlinearities

Yi Shen,* Chunmei Liu,† and Hengzhang Hu‡
*Harbin Institute of Technology,*
*150001 Harbin, People's Republic of China*

## Introduction

THE character of insensitivity to extraneous disturbance and internal parameter variations on a switching surface makes variable structure control (VSC) an effective method to control uncertain nonlinear dynamic systems. In theory, any control scheme that maintains its performance with the existence of uncertain factors should have a loop gain high enough. However, the inputs of a system are usually restricted by its physical structure and energy consumption, which also give the system inputs such nonlinear characters as saturation, deadzone, etc. The existence of nonlinear inputs is a source of degradation or, even worse, shows instability in the performance of the system.[1] Therefore, it is important to study the stability, robustness, and effective control of those systems with nonlinear inputs. In addition to nonlinear inputs, there are still plant uncertainties, such as identification inaccuracy, model truncation, plant parameter variation, etc. Therefore, more consideration is given to those systems with nonlinear inputs and uncertainties.

In Ref. 1, a new sliding mode control law based on the measurability of all of the system states is presented to ensure the global reaching condition of the sliding mode for uncertain systems with a series of nonlinearities. However, there often exist immeasurable states in a real system. Thus, we suggest a new law that requires only output information for output feedback VSC based on Ref. 1. An example is given to verify the sliding mode controller developed.

## Problem Formulation

As in Ref. 1, an uncertain system with nonlinear input is described as

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\Phi[\boldsymbol{u}(t)] + \boldsymbol{f}(\boldsymbol{x}, \boldsymbol{p}, t) \tag{1}$$

where $\boldsymbol{x}(t) \in R^n$, $\boldsymbol{u}(t) \in R^m$, $\boldsymbol{p}(t) \in R^q$, and $\boldsymbol{f}(t) \in R^s$ are the state variable, control input, uncertain parameter, and uncertain part of the system, respectively. $\boldsymbol{A} \in R^{n\times n}$ is the state matrix, $\boldsymbol{B} \in R^{n\times m}$ the input matrix, and $\Phi(\boldsymbol{u}) \in \Psi$ the nonlinear input function. It is also assumed that, for any initial condition $\boldsymbol{x}(t_0) = \boldsymbol{x}_0 \in R^n$, parameter $\boldsymbol{p}(t) \in R^q$, and control input $\boldsymbol{u}(t) \in R^m$, there exists a unique $\boldsymbol{x}(t; \boldsymbol{x}_0, \boldsymbol{p}, \boldsymbol{u})$ that satisfies the system described in Eq. (1).

To delineate the nonlinear input function $\Phi(\boldsymbol{u})$, some of the definitions in Ref. 1 are given here.

*Definition 1:* Let diagonal matrix $\Gamma = \mathrm{diag}[r_1, \ldots, r_m] \in R^{m\times m}$ be positive definite.

*Definition 2:* The allowed series nonlinearities $\Phi(\boldsymbol{u})$ belong to the set

$$\Psi = \big\{\Phi : R^m \to R^m;\, r_i\boldsymbol{u}_i^2 \le \boldsymbol{u}_i\Phi_i(\boldsymbol{u}),\, \boldsymbol{u}_i \in R,\; i = 1, \ldots, m,\, \boldsymbol{u} \in R^m\big\} \tag{2}$$

For the nonlinearity $\Phi(\boldsymbol{u}) \in \Psi$, $\Phi(\boldsymbol{u})$ satisfies

$$\boldsymbol{u}^T\Phi(\boldsymbol{u}) \ge \boldsymbol{u}^T\Gamma\boldsymbol{u} \ge r\boldsymbol{u}^T\boldsymbol{u} \tag{3}$$

where $r = \min\{r_1, \ldots, r_m\}$. For system (1) the following assumptions are made throughout.

*Assumption 1:* For all $i = 1, \ldots, m$, if $\boldsymbol{u}_i = 0$, then $\Phi_i(\boldsymbol{u}) = 0$.

*Assumption 2:* For the uncertain system (1), matrix pair $(\boldsymbol{A}, \boldsymbol{B})$ is controllable, and the system is observable.

*Assumption 3:* For the uncertain part of the system, $\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{p}, t)$ meets the following matching condition:

$$\boldsymbol{f}(\boldsymbol{x}, \boldsymbol{p}, t) = \boldsymbol{B}\xi(\boldsymbol{x}, \boldsymbol{p}, t), \qquad \|\xi(\boldsymbol{x}, \boldsymbol{p}, t)\| < k(\boldsymbol{y}, t)\quad \forall(\boldsymbol{x}, \boldsymbol{p}, t) \in R^n \times R^p \times R \tag{4}$$

## Design of Output Feedback Variable Structure Controller

Define the output of the system as $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$, $\boldsymbol{y} \in R^r$. Assume that the system is observable and controllable, $\boldsymbol{CB}$ is full rank, and the open-loop system is minimum phase.[2] Define the linear switching surface as

$$\boldsymbol{S} = \boldsymbol{G}\boldsymbol{y} \tag{5}$$

where $\boldsymbol{G} \in R^{m\times r}$. The sliding mode found from the equivalent control method is given by

$$\dot{\boldsymbol{x}} = [\boldsymbol{A} - \boldsymbol{B}(\boldsymbol{GCB})^{-1}\boldsymbol{GCA}]\boldsymbol{x} \tag{6}$$

Switching surface $\boldsymbol{G}$ is assumed to have already been chosen such that $\boldsymbol{GCB}$ is invertible and Eq. (6) has desired characteristics (see Ref. 3). The equivalent control $\Phi_{eq}$ in the sliding mode can be derived from $\dot{\boldsymbol{S}} = 0$:

$$\Phi_{eq} = -(\boldsymbol{GCB})^{-1}\boldsymbol{GC}(\boldsymbol{AX} + \boldsymbol{f}) \tag{7}$$

Note that the equivalent control $\Phi_{eq}$ is a mathematical tool derived for the analysis of sliding motion rather than a real control law that can be generated in practical systems. The equivalent control generates an ideal sliding motion on the switching surface. It can be seen that uncertain systems possess the same properties in the sliding mode irrespective of whether or not there are input nonlinearities. Thereby, the switching surface can be selected in the same way as the design for systems with linear input.[1]

Once a proper switching plane has been chosen, it is necessary to choose a discontinuous control law to guarantee the reaching condition of the sliding mode. The lemma in Ref. 1 is adopted here.

*Lemma 1:* For all allowable nonlinearities $\Phi_i$ belonging to set $\Psi$ in Eq. (2), there exists a known continuous function $\rho(\cdot) : R_+ \to R_+$, $\rho(0) = 0$, $\rho(\eta) \ge 0$ for $\eta \ge 0$, such that, for all $\boldsymbol{u} \in R^m$,

$$\rho(\|\boldsymbol{u}\|) \le \boldsymbol{u}^T\Phi(\boldsymbol{u}) \tag{8}$$

and there also exists a continuous function $\phi(\cdot) : R_+ \to R_+$, $\phi(0) = 0$, $\phi(\eta) \ge 0$, for $\eta \ge 0$, such that, for all $q \ge 0$,

$$\rho[\phi(q)] \ge q\phi(q) \tag{9}$$

*Proof:* Choose $\rho(\eta) = r\eta^2$, $r > 0$, then

$$\rho(\|\boldsymbol{u}\|) = r\|\boldsymbol{u}\|^2 = r\boldsymbol{u}^T\boldsymbol{u} \le \boldsymbol{u}^T\Phi(\boldsymbol{u}) \tag{10}$$

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*Professor, Department of Control Engineering, School of Astronautics; sy@ac411.hit.edu.cn.

†Doctoral candidate, Department of Control Engineering, School of Astronautics; rock@ac411.hit.edu.cn.

‡Professor, Department of Control Engineering, School of Astronautics.

Next, take $\phi(q) = \delta q$, $\delta \geq 1/r$, then

$$\rho[\phi(q)] = r\phi^2(q) = r \cdot \delta q \cdot \phi(q) \geq q\phi(q) \tag{11}$$

To obtain asymptotic stability of the sliding mode and to guarantee the reaching condition of the sliding mode, a VSC is presented:

$$\boldsymbol{u}(t) = -\frac{\boldsymbol{B}^T \boldsymbol{C}^T \boldsymbol{G}^T \boldsymbol{S}(t)}{\|\boldsymbol{B}^T \boldsymbol{C}^T \boldsymbol{G}^T \boldsymbol{S}(t)\|}\phi(q) \tag{12}$$

where $\phi(q) = \delta q$, $\delta \geq 1/r$, and function $q(\cdot) : R^n \times R \rightarrow R$ is defined as

$$q(\cdot) = \left(\frac{-\boldsymbol{S}^T \boldsymbol{GCBNy} + \alpha\|\boldsymbol{S}\|}{\|\boldsymbol{B}^T \boldsymbol{C}^T \boldsymbol{G}^T \boldsymbol{S}\|}\right) \tag{13}$$

Choose $\alpha$ such that $q(\cdot) \geq 0$. Note that $\|\boldsymbol{u}(t)\| = \phi(q)$, and the information required comes only from system output. The following theorem will prove that the developed control law (12) is capable of driving the uncertain system trajectory into the sliding mode $\boldsymbol{S}(t) = 0$.

*Theorem 1:* Consider the uncertain system (1) with series of nonlinearities subjected to Assumptions 1–3. If the input $\boldsymbol{u}(t)$ is defined by Eq. (12), then the sliding motion of the system trajectory is guaranteed reachable.

*Proof:* The reaching condition of the sliding mode is defined as $\boldsymbol{S}^T(t)\dot{\boldsymbol{S}}(t) < 0$. Substitute Eq. (1) into the time derivative of Eq. (5); we have

$$\begin{aligned}\boldsymbol{S}^T(t)\dot{\boldsymbol{S}}(t) &= \boldsymbol{S}^T \boldsymbol{GC}\dot{\boldsymbol{x}} = \boldsymbol{S}^T \boldsymbol{GC}(\boldsymbol{Ax} + \boldsymbol{B}\Phi(\boldsymbol{u}) + \boldsymbol{B}\xi) \\ &= \boldsymbol{S}^T \boldsymbol{GCAx} + \boldsymbol{S}^T \boldsymbol{GCB}\Phi(\boldsymbol{u}) + \boldsymbol{S}^T \boldsymbol{GCB}\xi\end{aligned} \tag{14}$$

From Eqs. (3) and (12), we get

$$\boldsymbol{u}^T\Phi(\boldsymbol{u}) = -\frac{\boldsymbol{S}^T \boldsymbol{GCB}}{\|\boldsymbol{B}^T \boldsymbol{C}^T \boldsymbol{G}^T \boldsymbol{S}\|}\phi(q)\Phi(\boldsymbol{u}) \geq r\boldsymbol{u}^T\boldsymbol{u} = \rho(\|\boldsymbol{u}\|) \tag{15}$$

Because $\|\boldsymbol{u}\| = \phi(q)$, from Lemma 1, we obtain

$$\boldsymbol{u}^T\Phi(\boldsymbol{u}) \geq \rho(\|\boldsymbol{u}\|) = \rho[\phi(q)] \geq q\phi(q) \tag{16}$$

Therefore, from Eqs. (15) and (16), the following expression holds:

$$\boldsymbol{S}^T \boldsymbol{GCB}\Phi(\boldsymbol{u}) \leq -q(\cdot)\|\boldsymbol{B}^T \boldsymbol{C}^T \boldsymbol{G}^T \boldsymbol{S}\| \tag{17}$$

Substitute Eq. (17) into (14), then

$$\boldsymbol{S}^T\dot{\boldsymbol{S}} \leq \boldsymbol{S}^T \boldsymbol{GCAx} + \boldsymbol{S}^T \boldsymbol{GCBNy} - \alpha\|\boldsymbol{S}\| + \boldsymbol{S}^T \boldsymbol{GCB}\xi \tag{18}$$

Equation (18) can be rewritten as $\boldsymbol{S}^T\dot{\boldsymbol{S}} \leq \boldsymbol{S}^T \boldsymbol{GC}(\boldsymbol{A} + \boldsymbol{BNC})\boldsymbol{x} - \alpha\|\boldsymbol{S}\| + \boldsymbol{S}^T \boldsymbol{GCB}\xi$. Note that a nonsingular transformation of switching surface will not change the sliding mode dynamics. If a particular switching surface $\boldsymbol{S}_1 = \boldsymbol{G}_1\boldsymbol{y}$ is chosen, it can be transformed to $\boldsymbol{S} = \boldsymbol{Gy}$, where $\boldsymbol{G} = (\boldsymbol{G}_1\boldsymbol{CB})^{-1}\boldsymbol{G}$. Therefore, without loss of generality, assume that $\boldsymbol{GCB} = \boldsymbol{I}$.

The reaching condition is defined as $\boldsymbol{S}^T\dot{\boldsymbol{S}} < 0$. For this system, it is

$$\boldsymbol{S}^T\dot{\boldsymbol{S}} \leq \boldsymbol{S}^T \boldsymbol{GC}(\boldsymbol{A} + \boldsymbol{BNC})\boldsymbol{x} - \alpha\|\boldsymbol{S}\| + \boldsymbol{S}^T \boldsymbol{GCB}\xi \leq 0 \tag{19}$$

Define the singular value decomposition of $\boldsymbol{GC}$ as $\boldsymbol{GC} = \boldsymbol{U}\Sigma \boldsymbol{V}_1^T$, where $\boldsymbol{U} \in R^{m\times m}$, $\Sigma \in R^{m\times m}$, and $\boldsymbol{V}_1 \in R^{n\times m}$. (The following proof is similar to the proof in Ref. 2.)

*Theorem 2:* For the control law in Eq. (12), choose

$$\boldsymbol{N} = -\left(\gamma\boldsymbol{I} + \boldsymbol{GCAV}_1\Sigma^{-1}\boldsymbol{U}^T\right)\boldsymbol{G} \tag{20}$$

where $\gamma > 0$ is a scalar. If $\alpha$ is chosen to satisfy $\alpha > (\|\boldsymbol{GCA}\|\Omega + k)$, $\Omega > 0$, and $q(\cdot) \geq 0$, then the reaching condition $\boldsymbol{S}^T\dot{\boldsymbol{S}} < 0$ is satisfied for $\{\boldsymbol{x} : \|\boldsymbol{x}_k\| \leq \Omega\}$.

*Proof:* Substitute $\boldsymbol{N}$ into the reaching condition in Eq. (19), and use $\boldsymbol{GCB} = \boldsymbol{I}$; we have

$$\boldsymbol{S}^T\dot{\boldsymbol{S}} \leq \boldsymbol{S}^T \boldsymbol{GC}\left(\boldsymbol{A} - \boldsymbol{BGCAV}_1\Sigma^{-1}\boldsymbol{U}^T\boldsymbol{GC}\right)\boldsymbol{x} - \gamma\boldsymbol{S}^T\boldsymbol{S} - \alpha\|\boldsymbol{S}\| + \boldsymbol{S}^T\xi \tag{21}$$

If $\boldsymbol{x}$ is decomposed as $\boldsymbol{x} = \boldsymbol{x}_k + \boldsymbol{x}_p$, where $\boldsymbol{x}_k \in \boldsymbol{N}(\boldsymbol{GC})$ and $\boldsymbol{x}_p \in \boldsymbol{N}^{\perp}(\boldsymbol{GC})$, then $\boldsymbol{GCx} = \boldsymbol{GCx}_p$. Simplify the first term on the right-hand side; it can be decomposed as

$$\begin{aligned}\boldsymbol{S}^T \boldsymbol{GCA}\left(\boldsymbol{I} - \boldsymbol{V}_1\Sigma^{-1}\boldsymbol{U}^T\boldsymbol{GC}\right)\boldsymbol{x} &= \boldsymbol{S}^T \boldsymbol{GCAx}_k \\ &+ \boldsymbol{S}^T \boldsymbol{GCA}\left(\boldsymbol{I} - \boldsymbol{V}_1\Sigma^{-1}\boldsymbol{U}^T\boldsymbol{GC}\right)\boldsymbol{x}_p\end{aligned} \tag{22}$$

If $\boldsymbol{GC} = \boldsymbol{U}\Sigma \boldsymbol{V}_1^T$, then $\boldsymbol{x}_p = \boldsymbol{V}_1\boldsymbol{\beta}$ for some vector $\boldsymbol{\beta}$. By the use of $\boldsymbol{V}_1^T\boldsymbol{V}_1 = \boldsymbol{I}$ and $\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$, the second term on the right-hand side of Eq. (22) is zero, and so the expression in Eq. (21) is reduced to

$$\boldsymbol{S}^T\dot{\boldsymbol{S}} \leq \boldsymbol{S}^T \boldsymbol{GCAx}_k - \gamma\boldsymbol{S}^T\boldsymbol{S} - \alpha\|\boldsymbol{S}\| + \boldsymbol{S}^T\xi \tag{23}$$

If $\Omega \geq \|\boldsymbol{x}_k\|$, the upper bound for the expression in Eq. (23) is

$$\boldsymbol{S}^T\dot{\boldsymbol{S}} \leq \|\boldsymbol{S}\|\|\boldsymbol{GCA}\|\Omega - \gamma\boldsymbol{S}^T\boldsymbol{S} - \alpha\|\boldsymbol{S}\| + \|\boldsymbol{S}\|k \tag{24}$$

Choose $\alpha$ such that $q(\cdot) \geq 0$ and $\alpha > (\|\boldsymbol{GCA}\|\Omega + \boldsymbol{k})$; then the reaching condition is satisfied.

If the system is strictly positive real, if $\gamma$ is chosen such that $2\gamma \geq -\lambda_{\min}(\boldsymbol{GCAV}_1\Sigma^{-1}\boldsymbol{U}^T + \boldsymbol{U}\Sigma^{-1}\boldsymbol{V}_1^T\boldsymbol{A}^T\boldsymbol{C}^T\boldsymbol{G}^T)$, and if the other parameters are chosen as they were earlier, the system is globally asymptotic (see the proof in Ref. 2).

## Numerical Simulations

For the nonlinear flexible structure, for example, the solar panel of a satellite, the vector equation of the dynamics can be written as

$$\boldsymbol{I}\dot{\omega} + \boldsymbol{B}_c^t\ddot{\boldsymbol{q}} + \boldsymbol{V}(\omega)\boldsymbol{H}_m = \boldsymbol{T}, \qquad \boldsymbol{A}_f\ddot{\boldsymbol{q}} + \boldsymbol{D}_f\dot{\boldsymbol{q}} + \boldsymbol{E}_f\boldsymbol{q} + \boldsymbol{B}_c\dot{\omega} = 0 \tag{25}$$

where $\boldsymbol{I}$ is the inertial matrix of the satellite and $\omega = [\omega_x \ \ \omega_y \ \ \omega_z]^T$ is the angle velocity matrix. $\boldsymbol{H}_m = [h_x \ \ h_y \ \ h_z]^T$ is the angle momentum of the satellite; $\boldsymbol{q} = [q_N \ \ q_P \ \ q_T]^T$ is the first mode of vertical, in-plane panel and torsion bending motion of the solar panel; and $\boldsymbol{T} = [T_x \ \ T_y \ \ T_z]^T$ is the vector of actuator torque. $\boldsymbol{A}_f$, $\boldsymbol{D}_f$, and $\boldsymbol{E}_f$ are the generalized mass, damp, and rigid matrix, respectively. $\boldsymbol{B}_c$ is the coupling matrix between the attitude of the satellite and the vibration of the solar panel.

Take the state vector as

$$\boldsymbol{x} = [\theta_x \quad \theta_y \quad \theta_z \quad q_N \quad q_P \quad q_T \quad \dot{\theta}_x \quad \dot{\theta}_y \quad \dot{\theta}_z \quad \dot{q}_N \quad \dot{q}_P \quad \dot{q}_T]$$

then Eq. (25) can be written as

$$\dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{B}\Phi(\boldsymbol{u}) + \boldsymbol{f}(\boldsymbol{x}, t) \tag{26}$$

where $\boldsymbol{x} \in R^n$, $\boldsymbol{A} \in R^{n\times n}$, $\boldsymbol{B} \in R^{n\times m}$, and $\boldsymbol{u} \in R^m$; the nonlinear term $\boldsymbol{f}(\boldsymbol{x}, t)$ is usually imprecise and is taken as the unknown disturbance and satisfies Eq. (4). In practical implementation, the control inputs are restricted by physical structure and energy consumption. Thus, the inputs in system realization always behave as bounded control. Here the nonlinearity in the input $\Phi(u)$ is taken as saturation whose saturated value is 80 N · m. The system satisfies all of the assumptions given earlier.

Where

$$\boldsymbol{V}(\boldsymbol{w}) = \begin{bmatrix} 0 & -w_z & w_y \\ w_z & 0 & -w_x \\ -w_y & w_x & 0 \end{bmatrix} \tag{27}$$

the parameters for the simulation are

$$\boldsymbol{I} = \begin{bmatrix} \boldsymbol{I}_x & \boldsymbol{I}_{xy} & \boldsymbol{I}_{xz} \\ \boldsymbol{I}_{xy} & \boldsymbol{I}_y & \boldsymbol{I}_{yz} \\ \boldsymbol{I}_{xz} & \boldsymbol{I}_{yz} & \boldsymbol{I}_z \end{bmatrix} = \begin{bmatrix} 3000 & 35 & 0 \\ 35 & 1500 & 0 \\ 0 & 0 & 2500 \end{bmatrix}$$

$$\boldsymbol{A}_f = \mathrm{diag}(a_N, a_P, a_T) = \mathrm{diag}(15.51, 19.92, 9.78)$$

$$\boldsymbol{D}_f = \mathrm{diag}(d_N, d_P, d_T) = \mathrm{diag}(0.01, 0.01, 0.01)$$

$$\boldsymbol{E}_f = \mathrm{diag}(e_N, e_P, e_T) = \mathrm{diag}(65.83, 708.94, 342.10)$$

$$\boldsymbol{B}_c = \mathrm{diag}(b_N, b_P, b_T) = \mathrm{diag}(145.00, 11.20, 84.00)$$

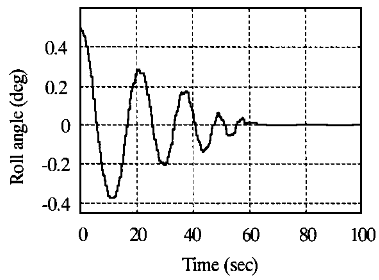


**Fig. 1 Angle response of the roll channel.**

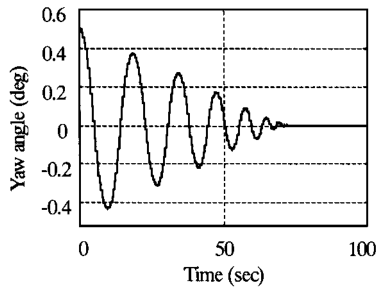


**Fig. 2 Angle response of the yaw channel.**

The angle responses of the roll channel and the yaw channel are given in Figs. 1 and 2. From the simulations we can see that the vibrations of the flexible structure attenuate asymptotically and the system is asymptotically stable. Without consideration of the nonlinearities in the input, the system is unstable.

## Conclusion

Based on the Ref. 1, a new output feedback VSC law is presented in which only the output information is required to guarantee the stability of the uncertain system with the nonlinearities in the input. An example is given to verify the sliding mode controller developed in this paper and to verify that the sliding mode is reachable for the given control law.

## References

[1]Hsu, K.-Ch., "Sliding Mode Controllers for Uncertain Systems with Input Nonlinearities," *Journal of Guidance, Control, and Dynamics*, Vol. 21, No. 4, 1998, pp. 666–669.

[2]Yallapragade, S. V., Heck, B. S., and Finney, J. D., "Reaching Conditions for Variable Structure Control with Output Feedback," *Journal of Guidance, Control, and Dynamics*, Vol. 19, No. 4, 1996, pp. 848–853.

[3]Edwards, C., and Spurgeon, S. K., "Sliding Mode Stabilization of Uncertain Systems Using only Output Information," *International Journal of Control*, Vol. 62, No. 5, 1995, pp. 1129–1144.